

FINAL EXAMINATION

26 June 2009 • Duration: 120 minutes

SUBJECT: CALCULUS I	
Head of Dept of Mathematics	Lecturers:
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INSTRUCTIONS:

Answer ALL questions in Section A and any TWO questions in Section B.

Calculators and dictionaries allowed. All other documents prohibited.

SECTION A

A1. [10 marks] Evaluate the following integrals:

$$(a) \int x \sin x dx, \quad (b) \int \frac{x+5}{(x-1)(x+2)} dx.$$

A2. [4 marks] If f' is continuous on $[a, b]$, show that

$$2 \int_a^b f(x)f'(x) dx = [f(b)]^2 - [f(a)]^2.$$

A3. [8 marks] State whether the following statements are true or false. If false, give an explanation or a counterexample. (If true, no explanation is required.)

(a) If f is continuous and positive on $[a, b]$ then $\int_a^b f(x)dx > 0$.(b) If f is continuous on $[a, b]$ and $\int_a^b f(x)dx > 0$ then $f(x) > 0$ for all x on $[a, b]$.

A4. [10 marks] Find the limit

$$\lim_{x \rightarrow 1} \frac{\int_1^{x^3} \cos(\pi t^2) dt}{x-1}. \quad [\text{Hint: Do not try to evaluate the integral.}]$$

A5. [10 marks] Use Newton's Method to find the root of the equation

$$x^3 + 3x + 1 = 0$$

in the range $-1 \leq x \leq 0$, correct to 5 decimal places. (Show full details of your calculations).A6. [10 marks] Determine the area of the region bounded by $y = 2x^2 + 10$ and $y = 4x + 16$.A7. [8 marks] Find the volume of the solid generated by rotating about the x -axis the region under the curve $y = e^x$ between $x = 0$ and $x = 1$.

*** PLEASE TURN OVER ***

SECTION B [Choose 2 out of 3 questions]

B1. On a certain day in Ho Chi Minh City, the temperature T (in $^{\circ}\text{C}$) could be modeled by the curve

$$T = 0.0004t^4 - 0.12t^2 + 33, \quad -12 \leq t \leq 12,$$

where t is the time in hours after noon. On this day ($-12 \leq t \leq 12$), find:

- (a) [10 marks] the average temperature,
- (b) [10 marks] the maximum and minimum temperatures and the times at which they occurred.

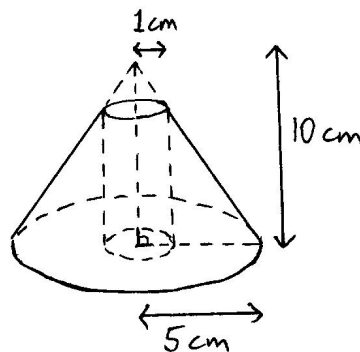
B2. (a) In a certain factory, the cost of producing the first 10 units is \$200 (fixed). After that, the marginal cost $C'(x)$, measured in dollars per unit, at an output level of x units is

$$C'(x) = \frac{1000}{x^2}.$$

- (i) [6 marks] Find the total cost of producing the 11th to 100th units.
 - (ii) [4 marks] Find the total cost of producing the first 100 units.
- (b) [10 marks] An athlete starts to run a race. The table below gives her speed during the first three seconds of the race, measured at half-second intervals. Estimate as accurately as possible the total distance that she traveled during these first three seconds. Explain clearly the method that you use.

time (s)	0.0	0.5	1.0	1.5	2.0	2.5	3.0
speed (m/s)	0.0	1.9	3.3	4.5	5.5	5.9	6.2

B3. (a) [10 marks] The pressure P and volume V of a sample of gas may both vary with time. However (under certain conditions) they always satisfy the equation $PV = C$, where C is a constant. Suppose that at a certain instant the volume is 600 cm^3 , the pressure is 150 kPa , and the pressure is increasing at a rate of 20 kPa/min . At what rate is the volume changing at this instant? Is it increasing or decreasing?



(b) [10 marks] A wooden children's toy is made by taking a solid right circular cone, of height 10 cm and base radius 5 cm, and drilling a hole of radius 1 cm down its central axis (as shown in the figure). Calculate the volume of the remaining solid.